Presume you are standing on a train station platform. Presume also that you are precisely in the middle of the platform. You are there to wave to your friend who will be travelling through the station on the 10:04 train. The train will not be stopping but your friend will be looking out the window for you. There is a storm brewing overhead but hopefully it holds off until after the train passes as you would prefer not to get wet.

At exactly 10:04, the train is passing through the station at a very high speed, nearing the speed of light (but not quite) and in an east to west direction. At the exact moment your friend on the train is precisely opposite where you are standing on the platform, two bolts of lightning strike – one at each end of the platform. Each of these bolts of lightning are precisely the same distance from you – one to the eastern end of the platform and the other at the western end. Thankfully you are unharmed, if not quite alarmed.

Did those two lightning strikes occur simultaneously? Explain your conclusion.

Let us examine the information given; at precisely 10:04 two strikes of lightning hit each end of the platform and light from them reaches you at precisely the same time, as they both struck equidistant from you and travelled at the same speed, you conclude the strikes occur simultaneously. You also conclude that, as your friend is traveling to the west, you will conclude that they will experience seeing the westerly most strike first. Your friend on the train, will in fact see the westerly strike first but, as they know that the speed of light is constant and that the strikes are equidistant from them, they must conclude that in fact, the westerly most strike happened first. So who is right? Well, according to Albert Einstein's theory of special relativity, both of them are right, in their own frame of reference. As the speed of light *is* constant for both of them, it is not meaningful to talk things happening simultaneously across separate frames of reference, this concept of simultaneity disappears when we talk about a reference frame dependant universe. When time is literally different in each frame, one can surely not talk about things happening at the 'same time' across frames. This is the fundamental bones of Einstein's theory that ultimately rewrote the rulebook on how we view space and time, but how does it work? How can it be that the time we experience is 'relative'; from our subjective viewpoint, time seems to be one of the only things that is constant; and an extraordinary claim like the one proposed in the theory of special relativity, would seem to need some extraordinary evidence to back it up. Luckily, that's exactly what Einstein and his contemporary's provide.

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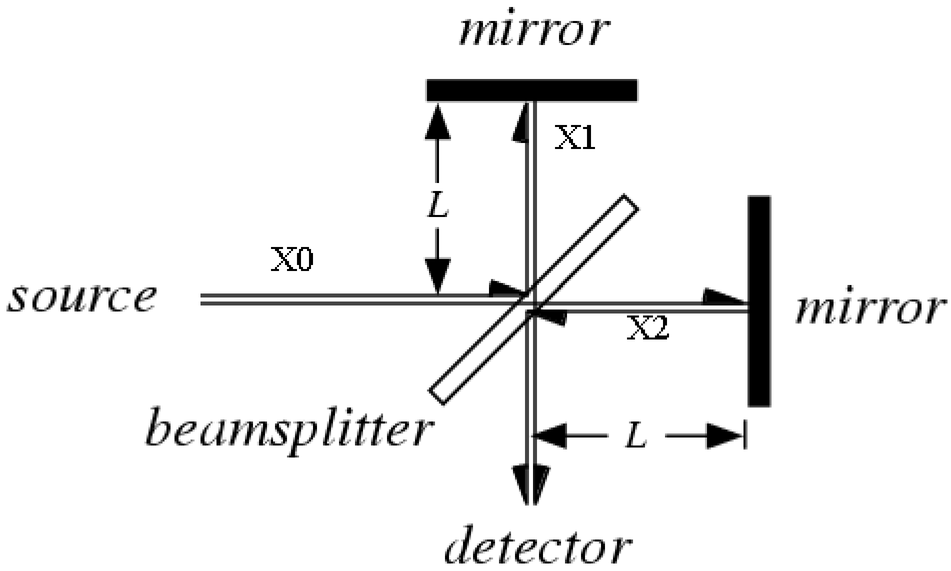
First we must show that the speed of light is constant in all reference frames. The following experiment was performed by Michelson and Morley in 1887, it was originally designed to work out the speed that the earth was moving though space, but the conclusions that it lead to had a far more profound impact on science than ever anticipated.

Assume you have a beam of light, x_0, that is split by a semitransparent mirror (called a beam splitter) such that half the light is reflected perpendicular to that of the incident ray (call this x_1) and half continues through the mirror unaffected (call this x_2)
X_1 and x_2 are then reflected directly back by mirrors, equidistant from the beam splitter. X_1 and x_2 will then be split again such that a composite beam perpendicular to x_0/x_2 is formed. As x_0 is being split and reformed, an interference pattern will be noted on observation of the reformed beam. The setup of the experiment is illustrated below.



As the earth is moving through space, if the speed of light was *not* constant it would be expected that as we rotate this setup relative to the direction of motion of the earth, we would expect different interference patterns, as, if the velocity of the earth is in the direction of x_0 then the speed of x_2 from the beam splitter will be C + V_{earth} and from the mirror to the beam splitter will be C- V_{earth}, therefore the time taken for a full reflection of $x_2 \neq$ time taken for a full reflection of x_1. This difference factor will vary as the experiment is rotated relative to the direction of motion of the earth, and thus the interference pattern will change.
However, what in fact Michelson and Morley observed, is that there was *no change* in the pattern, thus there could be no change in T_1 vs T_2, and if this were correct, then there were only two possible conclusions either that the earth is stationary in space, which is disprovable, or, *that the speed of light is constant in all reference frames.*

Given the results of the Michelson-Morley experiment, let us examine what this means for our thought experiment.

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Consider two inertial reference frames, (i.e. two reference frames that are not accelerating relative to each other, but have a net difference in velocity (V)). Now consider that in each reference frame there is a clock that is at some point in time synchronised with the clock in the other reference frame, such that at that point, they measure the exact same time.
Assume that you are in one reference frame (Y) and I am in the other (M), let any measurement of distance and time I make be denoted x and t respectably, and any measurement you make of distance and time be denoted x' and t'. Thus, if I were to measure the distance you travel relative to me in a given time, I could work out your velocity relative to me : $v = \frac{x}{t} \; or \; x = vt$

However, in your frame of reference you have not moved at all, and thus you would measure x'=0

Given $x - vt = 0$ and $x' = 0$
$=> \; x' = x - vt$

Given that both out clocks were synchronised at some point in time it would seem reasonable to assume t=t'

As the speed of light is constant in all frames of reference, I will measure $c = \frac{x}{t}$ and you will measure $c = \frac{x'}{t'}$ which can be written $x = ct$ and $x' = ct'$

Substituting this into our previous equation: $x' = x - vt$ we get $ct' = ct - vt$ which can be written ct'=t(c-v)

If it is the case that t'=t then this further simplifies to $c = c - v$ however, as Michelson and Morley proved, this is not the case. To rectify this error we must add an error term into the original equation(s) i.e. a factor by which our result is wrong.

Given $x' = x - vt \vdash x = x' + vt'$

Multiply both by the error term 'y'

$$=> x' = (x - vt)y, \qquad x = (x' + vt')y$$

Multiply together

$$=> x'x = (x - vt)y(x' + vt')y$$

$$=> x'x = y^2 (xx' + xvt' - vtx' - v^2tt')$$

As $c = \frac{x}{t} = \frac{x'}{t'}$ therefore $t = \frac{x}{c}$, $t' = \frac{x'}{c}$

Sub into above expression

$$=> x'x = y^2 (xx' + xv(x'c) - v\frac{x}{c}x' - v^2(\frac{x}{c})(x'/c))$$

As xx' exists in every term this expression can be simplified to:

$$1 = y^2 (1 + \frac{v}{c} - \frac{v}{c} - \frac{v^2}{c^2})$$

Which can be simplified to:

$$1 = y^2(1 - \frac{v^2}{c^2})$$

$$\text{Or } y^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\text{Therefore } y = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$$

This error term is known as the Lorentz transform.

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Whereas we originally wrote $x' = x - vt$, in fact, the relativistic correction for this equation is $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$. Obviously this will have some impact on what we consider to be simultaneous. Looking at this you will notice that if $v <<< c$ then very minimal change occurs to the result of the corrected equation compared to our original; as it will result in the error term being negligible.

Now consider the following reference frames y and m, for y measurements of space and time are made in x' and t'; and for m: x and t

Up until now we have assumed t=t',

Given $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$, and $x = ct,\ x' = ct'$

Substituting these into our relativistically correct formula for x' we get:

$$ct' = \frac{ct - \frac{vx}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Dividing through by c we get:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

= >

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

From this it is trivial that $t \neq t'$.

If we take a local measurement of time such that x=0, the equation further simplifies to

$$t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

And as $\sqrt{1-\frac{v^2}{c^2}} \in (0,1)$ i.e. it has a maximum value of 1 and a minimum value of 0, it is thus the case that $t \geq t'$ i.e. $t > t'$ for all $v > 0$.

This is certainly a remarkable result, as it implies that, dependant on the relative velocities of two reference frames, the *actual time* experienced in the reference frames will differ. This result fundamentally changes the way that we view the universe; it is no longer a meaningful question to ask "what is happening right now at a point in the universe" as the concept of "now" is no longer well defined and is dependent on the observers frame of reference. Similarly, when considering our original thought experiment, it is not meaningful to ask 'did the strikes occur simultaneously', as the concept of simultaneity is frame dependant. You on the platform, will experience the lightning strikes simultaneously, i.e. at the same instant in time; where as your friend will experience the westerly most strike first, with a distinct period of time passing before the easterly most; and the truly amazing thing is, you will both be right, the lighting strikes will both strike simultaneously *and* discretely depending on your frame of reference. It's all relative.

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